Ex 5 :

$$\begin{cases} x' = 4x - 2y \\ y' = x + y \end{cases} \quad \text{on pose : } A = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$$

$$X' = AX$$

$$\chi_A = X^2 - 5X + (4 - (-2)) = X^2 - 5X + 6$$

$$\Delta = 25 - 4(1)(6) = 25 - 24 = 1$$

$$r_1 = \frac{5-1}{2}, \; r_2 = \frac{5+1}{2}$$

$$r_1 = 2, \; r_2 = 3$$

$$\chi_A = (X-2)(X-3)$$

Soit $X = \begin{pmatrix} x \\ y \end{pmatrix} \in E_2(A)$

$$AX = 2X \Leftrightarrow \begin{cases} 4x - 2y = 2x \\ x + y = 2y \end{cases} \Leftrightarrow \begin{cases} x = y \\ y = y \end{cases}$$

$$E_2(A) = \text{Vect}\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$$

$$AX = 3X \Leftrightarrow \begin{cases} 4x - 2y = 3x \\ x + y = 3y \end{cases} \Leftrightarrow \begin{cases} x = 2y \\ y = y \end{cases}$$

$$E_3(A) = \text{Vect}\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right)$$

les sol de S_H sont de la forme :

$; \lambda, \mu \in \mathbb{R}$

$$X : t \mapsto A \begin{pmatrix} \lambda e^{2t} \\ \mu e^{3t} \end{pmatrix}$$

$$X : t \mapsto P \begin{pmatrix} \lambda e^{2t} \\ \mu e^{3t} \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R} \text{ et } P = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda e^{2t} \\ \mu e^{3t} \end{pmatrix}$$

$$= \begin{pmatrix} \lambda e^{2t} + 2\mu e^{3t} \\ \lambda e^{2t} + \mu e^{3t} \end{pmatrix}$$

Ex 1 :

1) $(E) : y' - \frac{x}{x^2-1} y = 2x$ sur $]1, +\infty[$

$(E_H) : y' - \frac{x}{x^2-1} y = 0$

$y_H : x \mapsto \lambda e^{\frac{1}{2}\ln(x^2-1)}, \quad \lambda \in \mathbb{R}$

$= \lambda \sqrt{x^2-1}$

on a : $y_0 : x \mapsto \lambda(x) \sqrt{x^2-1}, \quad \lambda \in \mathcal{C}^1(]1, +\infty[)$

Soit $x \in]1, +\infty[$

$y_0'(x) = \lambda'(x)\sqrt{x^2-1} + \lambda(x) \frac{2x}{2\sqrt{x^2-1}}$

y_0 est sol de (E) :

$$\lambda'(x)\sqrt{x^2-1} + \frac{\lambda(x)\, x}{\sqrt{x^2-1}} - \frac{x}{x^2-1} \lambda(x) \frac{\cancel{y_0(x)}}{\sqrt{x^2-1}} = 2x$$

$\Leftrightarrow \lambda'(x)\sqrt{x^2-1} = 2x \Leftrightarrow \lambda'(x) = \frac{2x}{\sqrt{x^2-1}}$